

**DYNAMICS OF FISH POPULATIONS DESCRIBED
BY RICKER'S MODEL AND ITS REORGANIZATIONS
UNDER THE WITHDRAWAL EFFECT**

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The central task of fishery is the maximal sustainable yield, i.e. an optimum withdrawal from the population with preservation of the maximal reproduction for a long time. It is considered that fishery gain is proportional to amount of the trade efforts. If the value of trade efforts could be determined by order, the task of optimization of withdrawal would be considered to definition of an optimum share of withdrawal and optimum amount of efforts ensuring the most equilibrium level of withdrawal. Fixing optimal efforts we could “fishing” and synchronous transfer a population in an optimum sustainable level. The basic difficulty here is that the trade efforts fail to be fixed. Their changes in time are defined by some factors such as weather conditions, technical equipment, and fishery stock. Apparently, dependence of amount of the trade efforts on an exploited population plays here the large role.

For illustration of this situation let us follow a mathematical designation. The relation between the number (abundance) of the consecutive generation is described by recurrent equation

$$X_{n+1} = aX_n e^{-bX_n} - \frac{f_n aX_n e^{-bX_n}}{m + f_n},$$

and in relative number

$$x_{n+1} = ax_n e^{-x_n} - \frac{f_n ax_n e^{-x_n}}{m + f_n} \quad \text{by } x_n = bX_n,$$

where x_n, x_{n+1} are the number of parents and recruits accordingly, a is a biotic potential, b is an ecological limitation, f_n is the fishery efforts in current year, m is the value of efforts ensuring half of maximum possible part of withdrawal. The first item is a reproduction (Ricker's curve) and the second item is the function of the withdrawal (annual catch).

For this model were obtained the tentative estimations of parameters on trade statistics data for real fish and crab populations. The values of annual catch and trade efforts were used as basic data designated accordingly as Y_n^* and f_n^* , and the model catches are equal

$$Y_n = \frac{kf_n ax_n e^{-x_n}}{m + f_n}, \quad \text{where } k=1/b.$$

The estimation of the model parameters consists in selection of such values of the parameters a, m, k and x_0 , by which the sequence Y_n approximates a known sequence Y_n^* in the best way. The values of trade efforts are defined as $f_n = f_n^*$. This problem was decided by a usual method of least squares. As a result is shown what exactly the offered function of the craft allows in the best way to describe the catch dynamic of trade populations.

With estimated parameters it was evaluated the trade stock of four exploiting objects in each year of fishery. Also, by this data rows it was concretized the statistical dependence between trade efforts amount (f) and value of current stock ($F(x)$) or population size before the reproduction (x). It was shown that the efforts in most cases regular changed when population size increases and this change is approximate by power function $f = \gamma[F(x)]^\alpha$ or $f = \gamma X^\alpha$ sufficiently

exact (the value of the determinant index is $0,62 < R^2 < 0,93$). The α parameter reflects the intensity of changes of efforts. As rule the effort amount increases with the degreasing of the population number. In this case the α parameter is negative. It is connected probably with such fact that fisher increases the trade efforts by small population size to ensure the required catch.

Conditions of the stability \bar{x}_M

$f = \gamma [F(x)]^\alpha$	$f = \gamma x^\alpha$
$-\frac{1}{1-\bar{x}_M} = \alpha_1 < \alpha < \alpha_2 = \frac{2-\bar{x}_M}{\bar{x}_M(1-\bar{x}_M)}$	$-1 = \alpha_1 < \alpha < \alpha_2 = \frac{2}{\bar{x}_M} - 1$
$m_0 = \left(\frac{\bar{x}_M}{1-\bar{x}_M} \right)^{\alpha-1}$	$m_0 = (1-\bar{x}_M) \bar{x}_M^{\alpha-1}$

The α parameter is an “indicator” of the fishery effect. If α appears less α_1 then the number passes to a new stationary level meaning, apparently, the degeneration of the population (zero equilibrium here is stable). If α appears more α_2 in the population there are **oscillations of number called by the craft from variable part of withdrawal.**

Now all models’ parameters are known and it is possible the results of commercial fishing to analyze theoretically and the state of the corresponding commercial populations to assess. The model curves, as well as the bisector of the first coordinate angle are shown in Figure 1. As large is the area of the figure limited by the model curve and bisector of the first coordinate angle as the balance between maximal yield and maximal reproduction is more stable (population of herring). In case of walleye pollack this area is very low, therefore, there is an actual hazard of commercial overfishing. If the model curve comes nearer or lays under bisector, the population will inevitably vanish as an object of commercial fishing (halibut, Baltic herring).

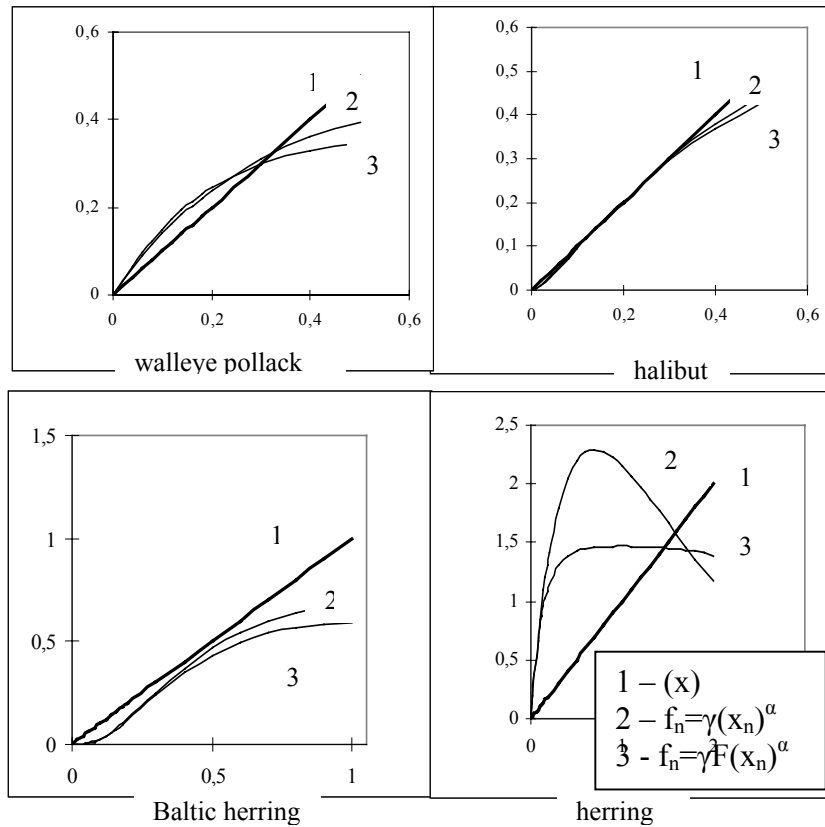


Figure 1. Simulation of the population dynamic

Thus, construction, analysis, and verification of a very simple mathematical model of population dynamics of commercial fish allows the following conclusion to be drawn: if fishing intensity is a nonlinear function of population size, fishing intensification inevitably causes either population size oscillations or population degeneration.

