CURIOSITY, BIFURCATION AND CHAOS:

A TRIBUTE TO BILL RICKER'S INQUIRING MIND

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EXTENDED ABSTRACT ONLY – DO NOT CITE

Bill Ricker retired from his position as scientist at the Pacific Biological Station in Nanaimo, British Columbia, three years before I began working there. Because he retained an office and used it frequently, I didn't realize at first that he was, in fact, retired. Although I knew of his reputation as an accomplished scientist, it took me many years to appreciate the scope of his achievements. I began working in fisheries as a naïve mathematician, with much to learn about biology and the broader world of scientific inquiry. Bill brought a compelling curiosity to any subject that interested him. For example, he introduced me to the controversies about crop circles with the investigative article by Nickell and Fischer (1992).

As a mathematician, I had the opportunity to participate in some of Bill's mathematical recreations. He would occasionally pass me articles of interest, such as Stewart's (1995) discussion of flower structures. What algorithm could possibly explain the beautifully tight packing of seeds on the head of a sunflower? A possible answer relates to:

- the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34,…, in which each number is the sum of the previous two,
- the golden number $\varphi = \frac{\sqrt{5}-1}{2}$ defined by the equal ratios $\frac{\varphi}{1} = \frac{1}{1+\varphi}$, and
- the golden angle $\theta = (1-\varphi)\times 360^\circ = 137.507764\cdots^\circ$.

Successive Fibonacci ratios tend to the golden number, that is $\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \frac{21}{34}, \cdots \to \varphi$, and the golden angle helps define the packing algorithm. To draw n points (x_k, y_k) within the unit circle, use the formulas

(1) $$x_k = \sqrt{k/n}\cos(k\theta),\ y_k = \sqrt{k/n}\sin(k\theta), \text{ where } k = 1,\ldots,n.$$

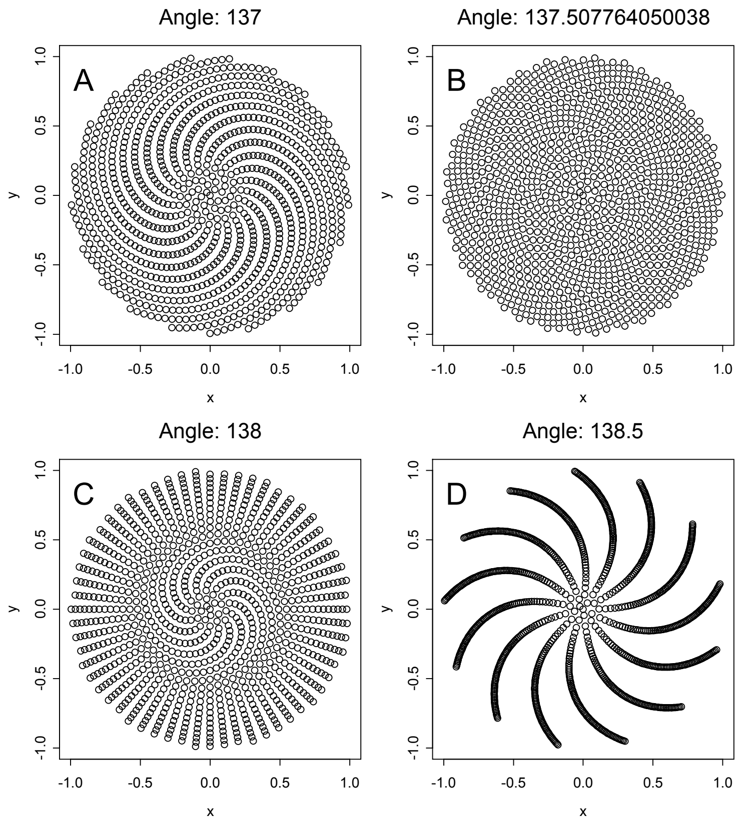

Figure 1. Patterns of $n = 1000$ points (x, y) produced by the algorithm (1) with various rotational angles θ. In panel B, θ is the golden angle $(1-\varphi)\times 360^\circ$.

In polar coordinates, each successive point is located by rotating through the angle θ and increasing the distance from the origin slightly, where the square root gives a radial distance that produces even spacing of points within the area. Figure 1B shows the outcome of the algorithm (1) with the golden angle θ. Nearby angles θ gives distinctly different results in Figs. 1A, 1C, and 1D. This example illustrates two important features of a chaotic process:

- a simple algorithm can produce a complex pattern, and
- a small change in the algorithm can produce a major change in the pattern.

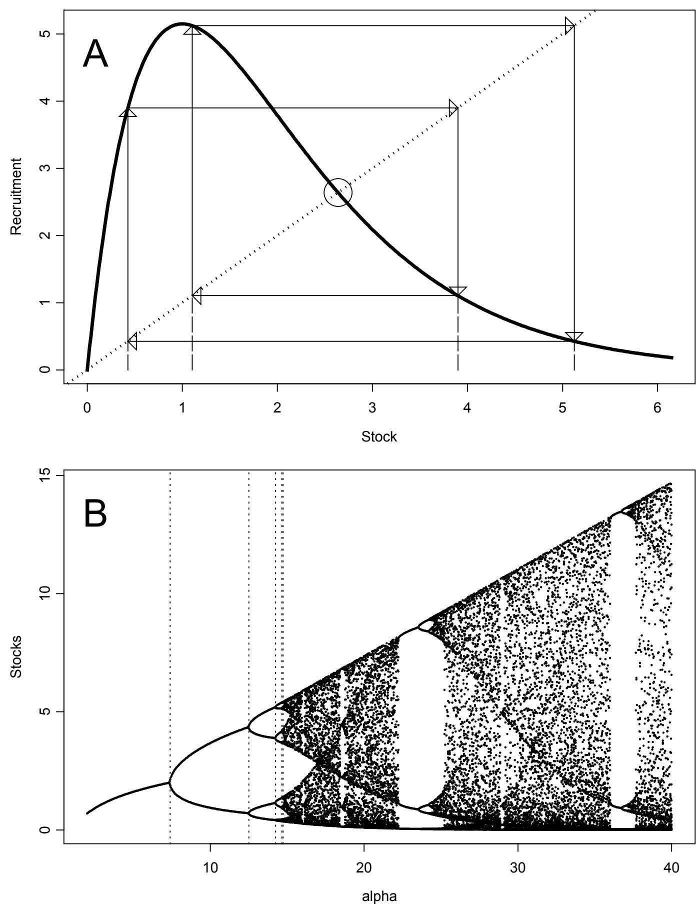

Figure 2. (A) Ricker curve with $\alpha = 14$ and $\beta = 1$ (thick solid line), showing the stable oscillating pattern among four stock sizes. Arrows indicate the progression from a stock S to a recruitment R and then to the next stock S, via the line $R = S$ (dotted). A circle marks the replacement point (2), which is not stable in this case. (B) Stable stock sizes in relation to α for a Ricker curve with $\beta = 1$. The initial line segment on the left corresponds to the single equilibrium value S_{eq} in (2) for $\alpha \le \alpha_1$.

Ricker's famous curve $R = \alpha S e^{-\beta S}$, which describes the relationship between stock S and subsequent recruitment R, also has interesting chaotic properties. In 1995, Bill asked me why his curve appeared as an example in a technical book by a Russian author (Kuznetsov 1995, p. 112). The answer relates to stable points in the implied population dynamics. If the stock replaces itself in each generation $(R = S)$, then Ricker's equation becomes $S = \alpha S e^{-\beta S}$, with the equilibrium solution

(2) $$S_{\text{eq}} = \frac{1}{\beta} \log \alpha .$$

In fact, this simple result fails to capture complex behaviour of the Ricker function, which depends on critical values of the parameter α given by

(3) $$\alpha_1 = e^2 = 7.38907\cdots,\ \alpha_2 = 12.50925\cdots,\ \alpha_3 = 14.24425\cdots,$$
$$\alpha_4 = 14.65267\cdots,\ \ldots .$$

The equilibrium value S_{eq} in (2) becomes unstable when $\alpha > \alpha_1$. The stock alternates between two equilibrium sizes when $\alpha_1 < \alpha \le \alpha_2$, and more generally oscillates through 2^k sizes when $\alpha_k < \alpha \le \alpha_{k=1}$. Figure 2A illustrates the oscillation among four values when $\alpha = 14$. The global relationship between α and multiple equilibria gives the remarkable pattern in Fig. 2B. Vertical dotted lines show the values α in (3) at which the number of stable points doubles. These are called "bifurcation" points for the parameter α. They follow a nearly geometric progression, so that the number of equilibria effectively becomes infinite and the Ricker curve produces "chaos" at values α somewhat higher than α_4. Still higher values α give additional regions of stability, bifurcation, and chaos.

When Bill proposed his recruitment model in 1954, it would have been impossible to foresee its complex consequences in Figure 2B. Computing technology has changed dramatically since then, and the link between algorithms and patterns has become a hot topic in modern science. While I prepared this talk, Wolfram (2002) published a lengthy book claiming that this link offers a whole new approach to the analysis of complex systems. The example in Fig. 1 above plays a role in his argument (op. cit., p. 411). Waldrop (1992) further documents the development of this approach in his account of the founding of the Santa Fe Institute. Bill's career spanned a period of great technological change, but he retained a genuine spirit of inquiry to the end. His interests in the structure of sunflowers (Fig. 1) and the chaotic properties of recruitment curves (Fig. 2) illustrate his lifelong fascination with creative ideas.

Acknowledgements

Bill had a great interest in Russian contributions to science, and perhaps he discovered the reference to Kuznetsov (1995) in that context. Similarly, Kuznetsov might have noticed the Ricker curve because of Bill's strong scientific reputation in Russia. Either way, Kuznetsov's book makes a remarkable contribution to the literature on nonlinear dynamical models. The author wrote it while in Amsterdam, but he cites work at the Research Computing Centre of the Russian Academy of Sciences, which was renamed in 1992 as the Institute of Mathematical Problems in Biology.

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